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<u>Thm</u> (X,H) polarized K3.

T_X is μ_H-stable $\left(\Leftrightarrow \nexists \underset{\text{l.b.}}{\mathcal{L}} \subsetneq T_X \text{ of non-negative degree.}\right)$

Last time: If T_X contains a l.b. of pos. degree, then X is uniruled,

i.e. $\exists\, \underset{\text{curve}}{D} \times \mathbb{P}^1 \overset{\text{dominant}}{\dashrightarrow} X$.

<u>Lem.</u> A K3 surface X is not uniruled.

proof). Assume $\exists\, D \times \mathbb{P}^1 \dashrightarrow X$ dominant.

By resolving the indeterminancy, we obtain a dominant morphism $f: Y \to X$.

By generic smoothness, f is generically étale. Take $\mathcal{U} \subseteq Y$ open so $f|_{f^{-1}(\mathcal{U})}$ is étale

We want to show $H^0(X, \omega_X) \to H^0(Y, \omega_Y)$ is injective.

It suffices to show $H^0(\mathcal{U}, \omega_X|_{\mathcal{U}}) \hookrightarrow H^0(f^{-1}(\mathcal{U}), \omega_Y|_{f^{-1}(\mathcal{U})})$ is injective.

Now, the following two lemmas suffice.

<u>Lem</u>

$f: Y \to X$ étale $\rightsquigarrow f^*\omega_X \cong \omega_Y$. ($\because f^!\omega_X = \omega_Y$ in general and $f^! = f^*$ if f étale)

<u>Lem</u>

$f: Y \to X$ dominant, $\mathcal{L}$ l.b. on X $\rightsquigarrow H^0(X, \mathcal{L}) \to H^0(Y, f^*\mathcal{L})$ is injective.

proof). f maps the generic pt to the gen. pt and since non-zero global section cannot vanish at the generic pt. ($\because \Gamma(X, \mathcal{L}) \hookrightarrow \mathcal{L}_\eta$ is inj. e.g. Liu) $\square$

Now, since $h^0(\omega)$ is birat. invariant (e.g. Har. II.8.19), $h^0(\omega_Y) = h^0(\mathbb{P}^1 \times D) \overset{\text{Kunneth}}{=} 0$

while $h^0(\omega_X) = 1$, which is absurd. $\square$

<u>Cor.</u> T_X does not contain a l.b. of positive degree.

proof of Thm)

By Cor., it suffices to show $\mathcal{T}_X$ does not contain a l.b. of deg 0.

First, assume $\mathcal{T}_X$ contains $\mathcal{O}_X(D) \not\cong \mathcal{O}_X$ of degree 0, i.e, $D.H=0$

Take $n \gg 0$ so $-D+nH$ is ample. By the Hodge index thm, $D.(-D+nH) = -D^2 > 0$, which is absurd by the Corollary.

Next, assume $\mathcal{O}_X \subseteq \mathcal{T}_X$. By the Hodge theory, we have $h^0(\mathcal{T}_X)=0$, but $h^0(\mathcal{O}_X)=1$, which is absurd $\square$